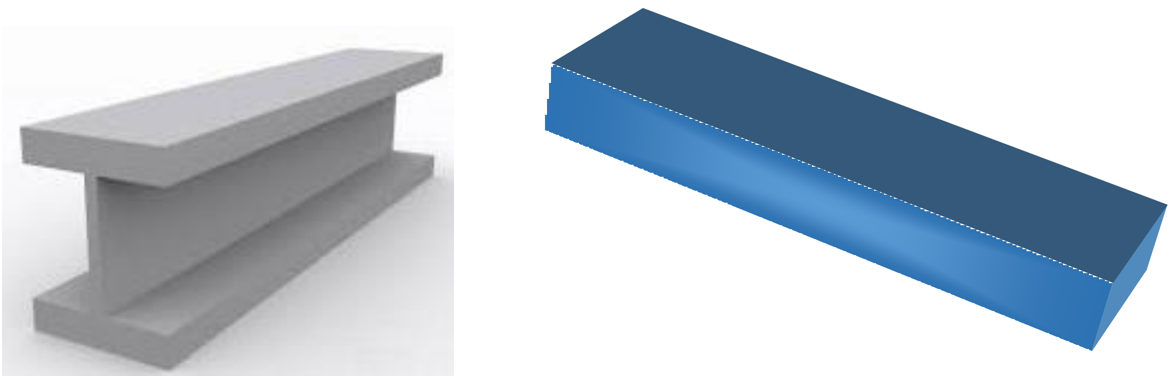
Syllabus: ***Free body diagrams; Examples on modeling of typical supports and joints and discussion on the kinematic and kinetic constraints that they impose.***

A **free body diagram** or FBD is a graphical illustration used to visualize the applied forces, moments, and resulting reactions on a body in a given condition. It shows a body with all the applied forces, moments, and reactions, which act on the body. The body may be a single one like a beam or bar or be consist of multiple internal members such as truss as shown in figure 1 and figure 2. A number of free body diagrams may be necessary to solve a complex problem.



(a) (b)

Fig. 1 Beam (a) I-section beam (b) Rectangular section beam

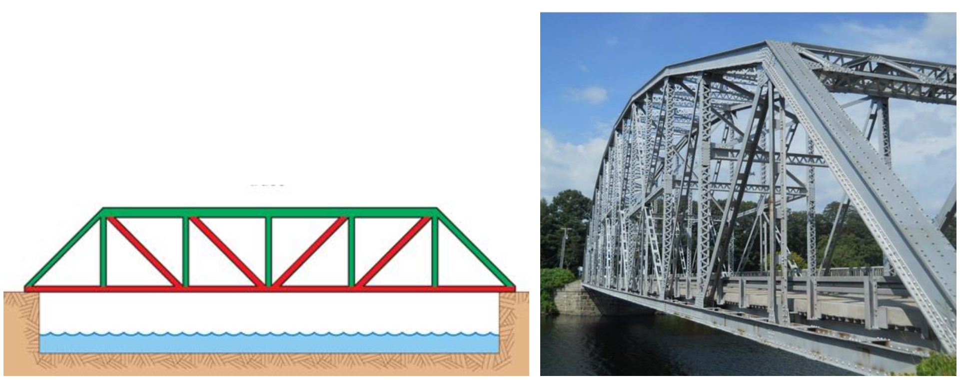


Fig. 2 Truss

Prepared by: Dr. B. K. Biswas

Free body diagrams are used to visualize the forces and moments applied to a body and to calculate the resulting reactions in different types of mechanics problems. It is used both to determine the loading of individual structural components and to calculate internal forces within the structure.

Some of the **Key Features of FBD** are

- Here we will use a simplified version of the body like often a dot, beam or a box
- Forces shown as straight arrows pointing in the direction they act on the body
- Moments shown as curved arrows pointing in the direction they act on the body
- Frequently reaction to applied forces are shown with hash marks through the stem of the arrow
- A coordinate system

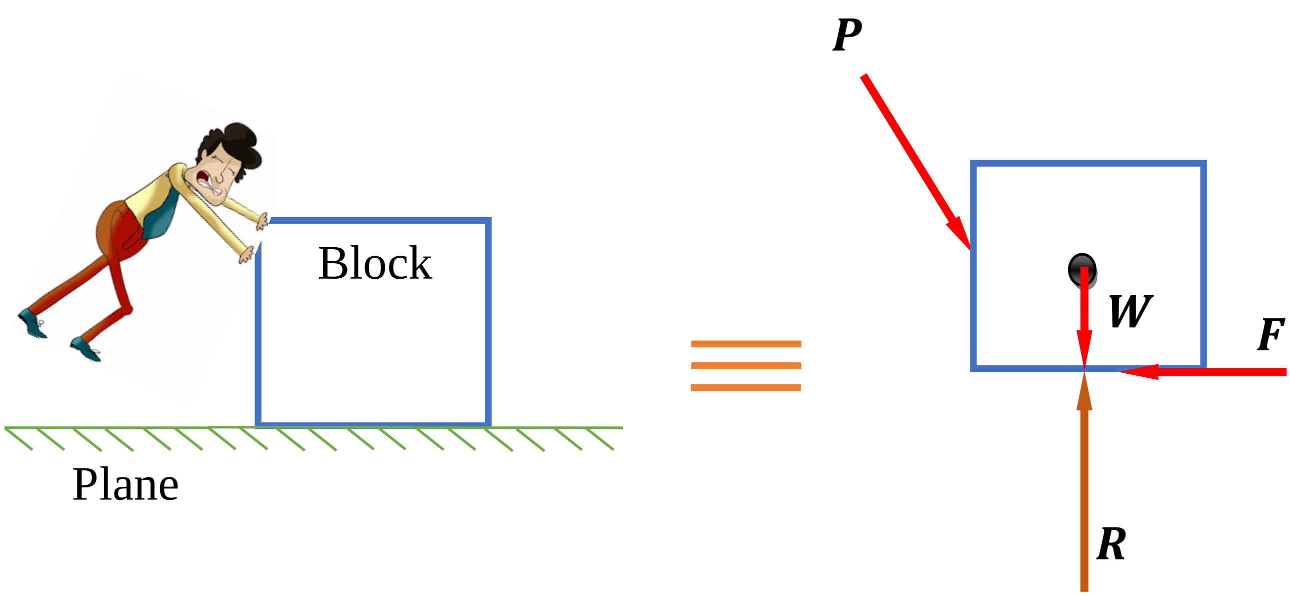



Fig. 3 (a) Physical condition (b) FBD

These features can be explained with the help of an example. Let us consider a block of weight W is on a horizontal plane and a man is pushing the block with force P as in Fig. 3(a). Here the block is the body. In the FBD there will be the concern body and all acting force and moment acting on the body. It needs to replace all physical contacts by forces. The man is applying a force P to the body so man is replaced by P and the horizontal plane is replaced by the reaction force R. The other forces working on the body are friction force, F and the weight of the body, W. So we are having the FBD as shown in the figure (b).

Under the action of a force if a body is tends to rotate then it is considered that a moment is working on the body. The moment of force is the cross product of distance, r and force, F that causes the moment. The moment is denoted by the curved arrows pointing in the direction they act on the body as shown in the figure 4.

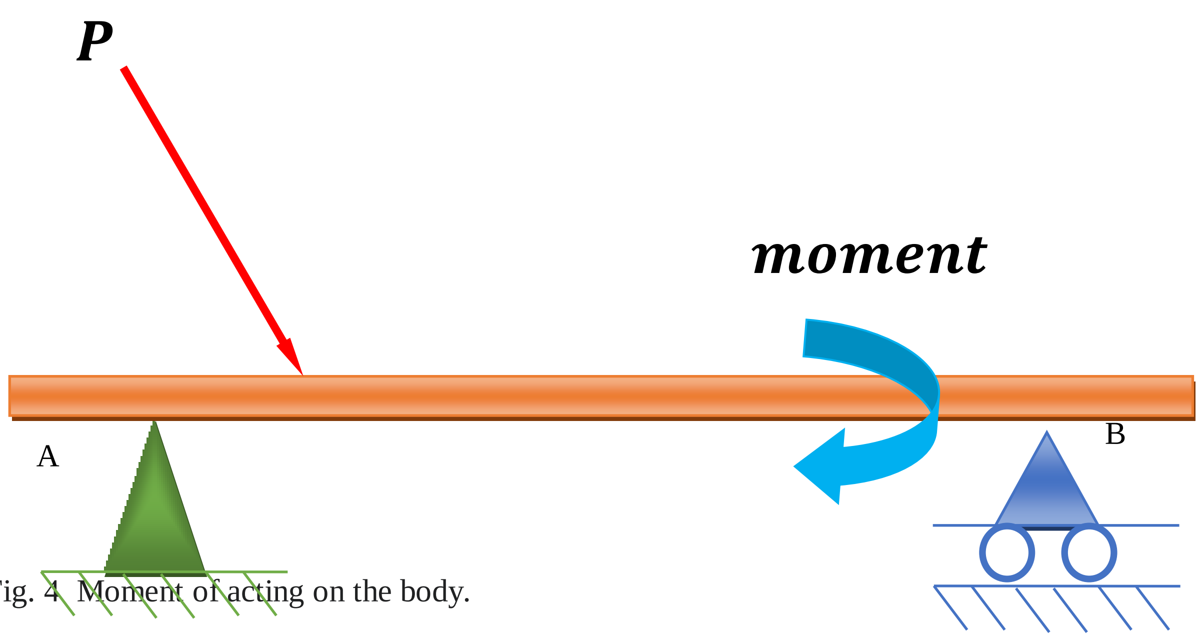



Fig. 4 Moment of acting on the body.

**Modeling the Body**

A body may be assumed in three ways:

- ***Particle***
- ***Rigid Body and***
- ***Non-rigid body***

If the mass and dimension of the body is small and any rotational effects are zero or have no interest then it is said to be a particle.

If the shape and geometry of the body does not change or changes are of no interest with the effect of the working forces of the body then body is called rigid body.

On the other hand if the change of shape and geometry is significant then the body is Non-rigid body

# Basic Kinematics and Kinetics Constraint of Rigid Bodies

Kinematics describes the motion of points, bodies, and systems of bodies without considering the forces that cause them to move. Motion means we will study the displacement, velocity and acceleration.

Kinetics describes the motion of points, bodies, and systems of bodies with considering the forces that cause them to move. Here including displacement, velocity and acceleration we will also study the forces or moment that cause the motion.

Constraint means prevention of motion. It can be explained by degree of freedom. The ***degrees of freedom*** (DOF) of a rigid body is defined as the number of independent movements it has. Figure 5 shows a rigid body in a plane. To determine the DOF of this body we must consider how many distinct ways the bar can be moved. In a two dimensional plane such as this computer screen, there are 3 DOF. The bar can be *translated* along the *x* axis, translated along the *y* axis, and *rotated* about its centroid.

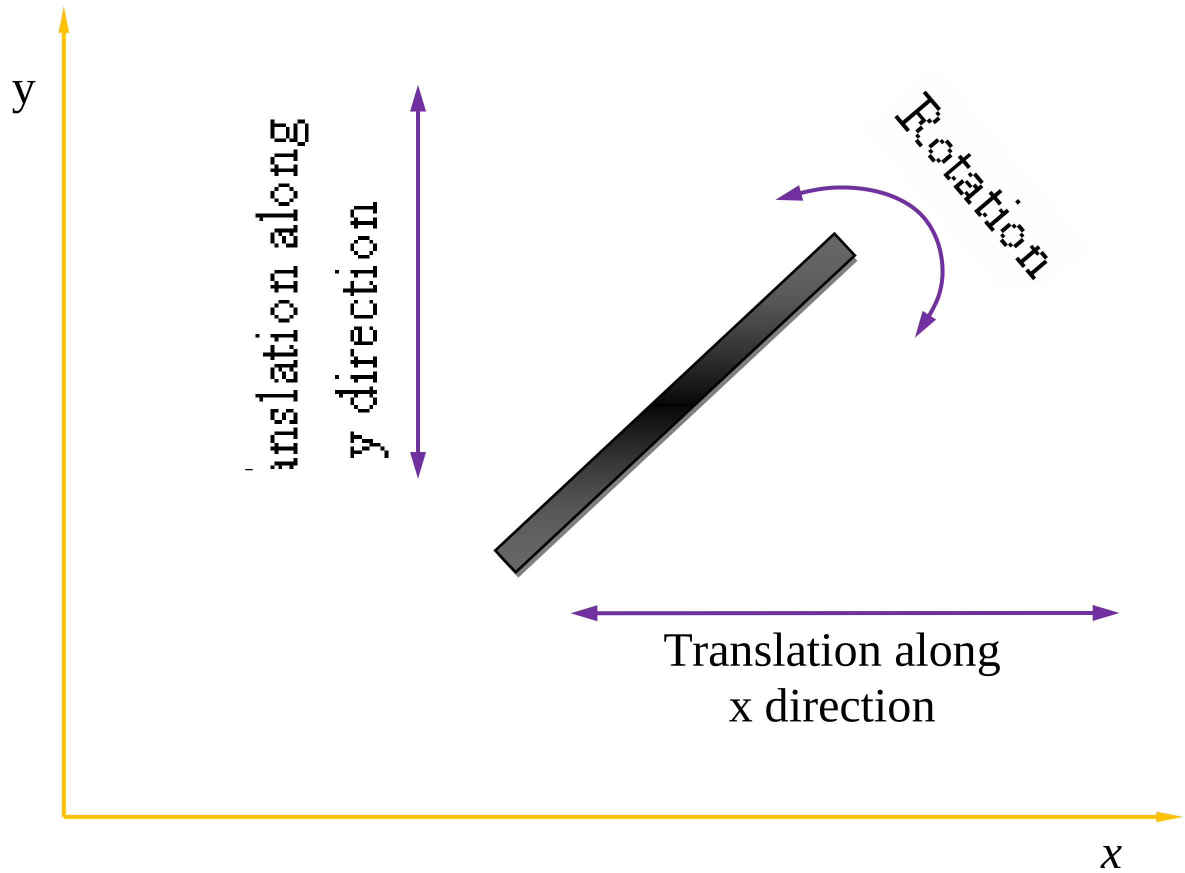



Fig. 5 ***Rigid body in a plane***

A support is constraints of a rigid body motion. Kinetic constraints means the reaction forces and moment that imposed by the support. To obtain the reaction forces and moment we need to understand the condition of static equilibrium.

## Condition of static and dynamic equilibrium

The number of forces and moments shown in a free body diagram depends on the specific problem and the assumptions made. In statics all forces and moments must balance to zero. It can be explained with the help of the example figure 4. Let us draw the FBD of the problem as shown in Fig. 5.

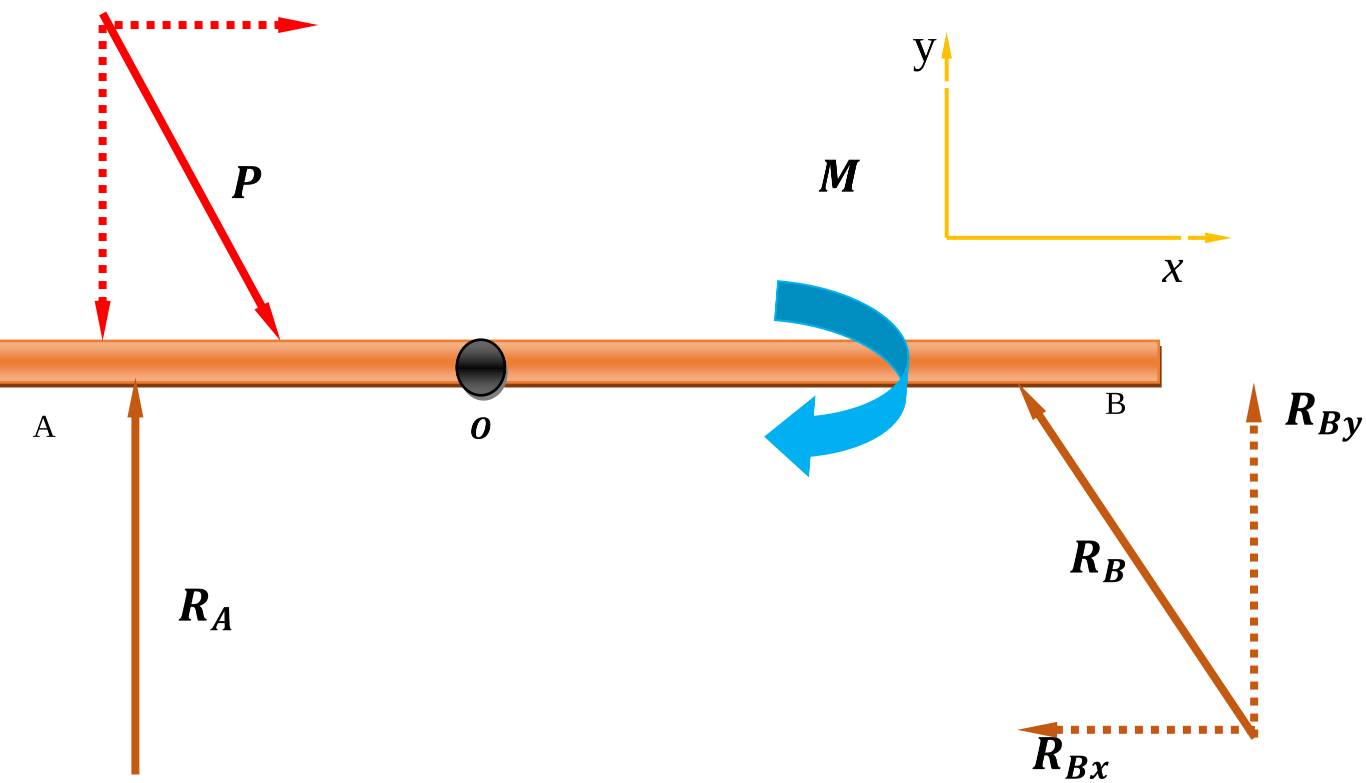



Fig. 5 Force resolution in a FBD

Two supports are replaced by two reaction force $R_A$ and $R_B$. Now a coordinate system is required to solve this problem which is shown in figure by Cartesian coordinate system. The forces need to resolve in X-direction and Y-direction. Here X or Y-directions are arbitrary but they always mutually perpendicular.

If the body is in static condition then applying Newton's law of motion, we have the condition of static equilibrium.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_O = 0$$

Here summation of $F_x$ means the summation of x-component of all forces working on the body. Similarly $F_y$ means the summation of y-component of all forces.

The third condition of static equilibrium is derived from moment equitation. The body is in static condition means no rotation. We know moment or couple is responsible for angular motion. So the summation of moment of all forces about any arbitrary point O is equal to zero.

If the body is not in static condition then we are getting condition of dynamic equilibrium.

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum M_O = I\alpha$$

Here, ax and ay are the linear acceleration in the direction of x and y respectively and α is the value of angular acceleration. If any one of the values, ax, ay or α is non-zero then the body is said to in dynamic condition.

## *Typical supports and joints*

A support is constraints of a rigid body motion. In a static structure a little deflection is required for a structure to protect other surrounding materials from those forces. Example of this deflection is shown in figure. F force is applied at the end of beam of length L then we will observe a deflection f. Generally is structural element like bridge, flyover, shaft of a motor this deflection is not visible but it plays an important role to protect the structure.

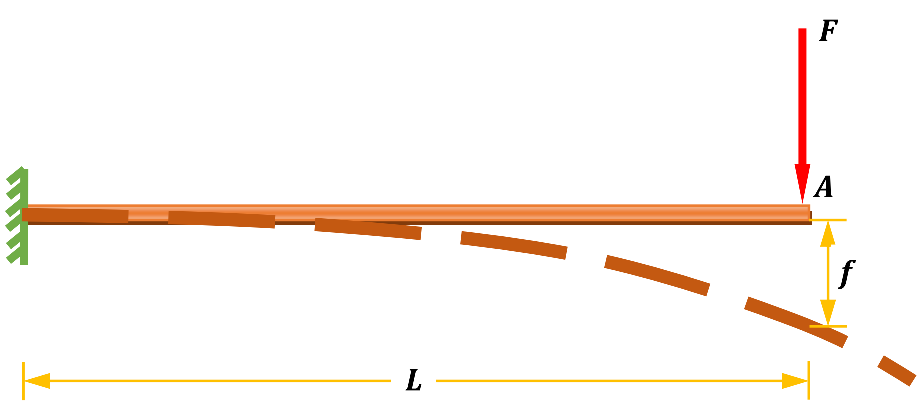


Fig. 6 Deflection of a beam under the action of a force F.

Based on the types of deflection there are 5 different types of supports.

- **Roller**,
- **Pinned**,
- **Fixed**,
- **Hanger** and
- **Simple support**.

Before starting support we need to understand about reaction force. From Newton's law of motion we know that every action has an equal and opposite reaction. The applied force may cause acceleration or movement or displacement. If there is any obstruction to move the body then reaction is working on the body. If the body is free to move in a direction then action produces no reaction but it displaces the body. Now let's discuss about the supports.

## Roller Supports

Roller support allows a limited amount of lateral movement. In response to a lateral force pushes on the structure, a structure on roller skates will roll away in response to the force. Let us consider an example shown in figure 7. A beam is supported by roller support. A force F is applied to the beam. F can be resolved by Fx and Fy. The beam is free to move under the action of Fx but unable to move under Fy. So there is a component of reaction $R_{Ay}$. The movement of the beam in x direction is not restricted so there is no reaction component in x-direction.

This support type is assumed to be capable of resisting normal displacement.

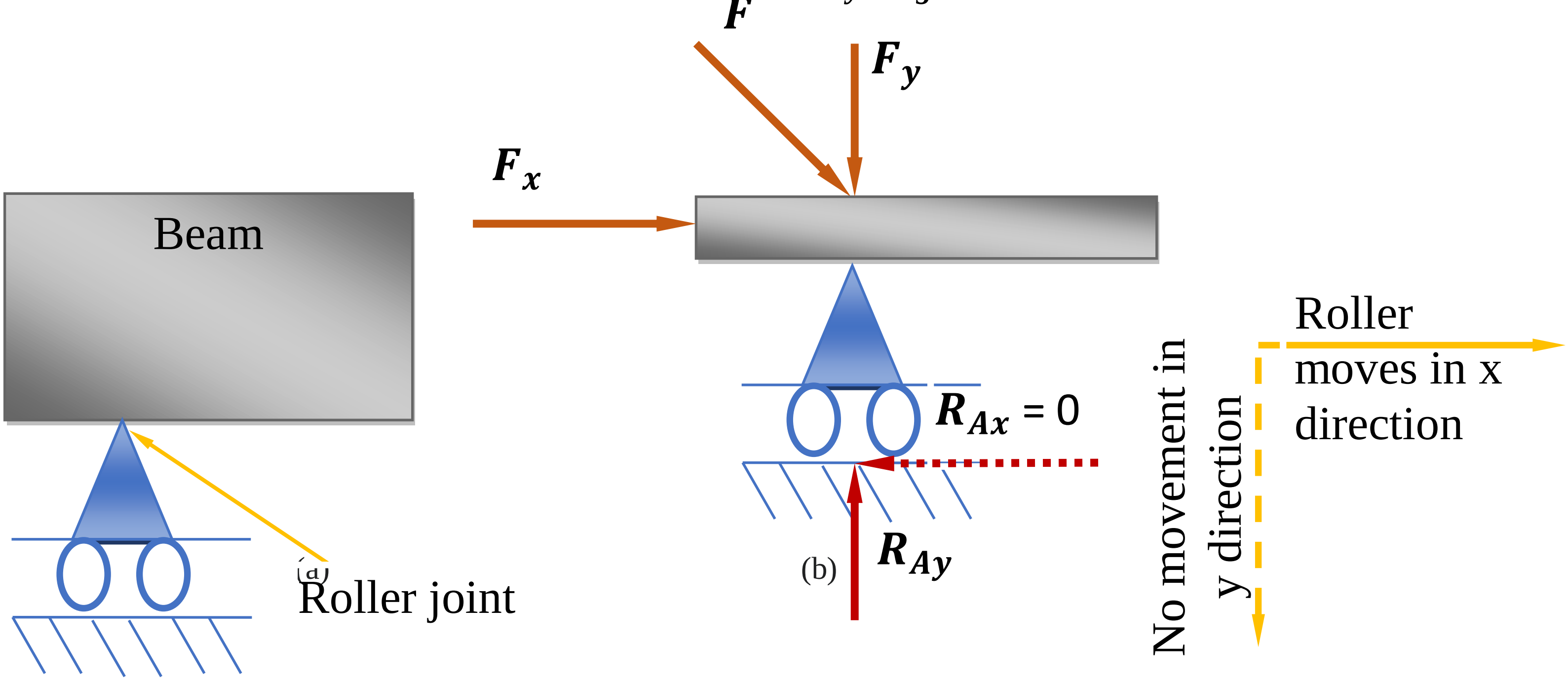



Fig. 7 Roller support: (a) schematic presentation and (b) FBD

No movement in y direction

Fig. 8 Roller support in application

The typical application of Roller supports is in large bridges. In civil engineering, roller supports can be seen at one end of a bridge.

## Pinned Support

A pin joint is a connection between two objects that allows only relative rotation about a single axis. Pin joint is followed in pinned support. A pinned support can resist both vertical and horizontal forces but not a moment as shown in Fig. 9. So, both reaction component $R_{Ax}$ and $R_{Ay}$ is present there.

This support will allow the structural member to rotate, but not to translate in any direction. Many connections are assumed to be pinned connections but in actual cases pinned support also resist a small amount of moment.

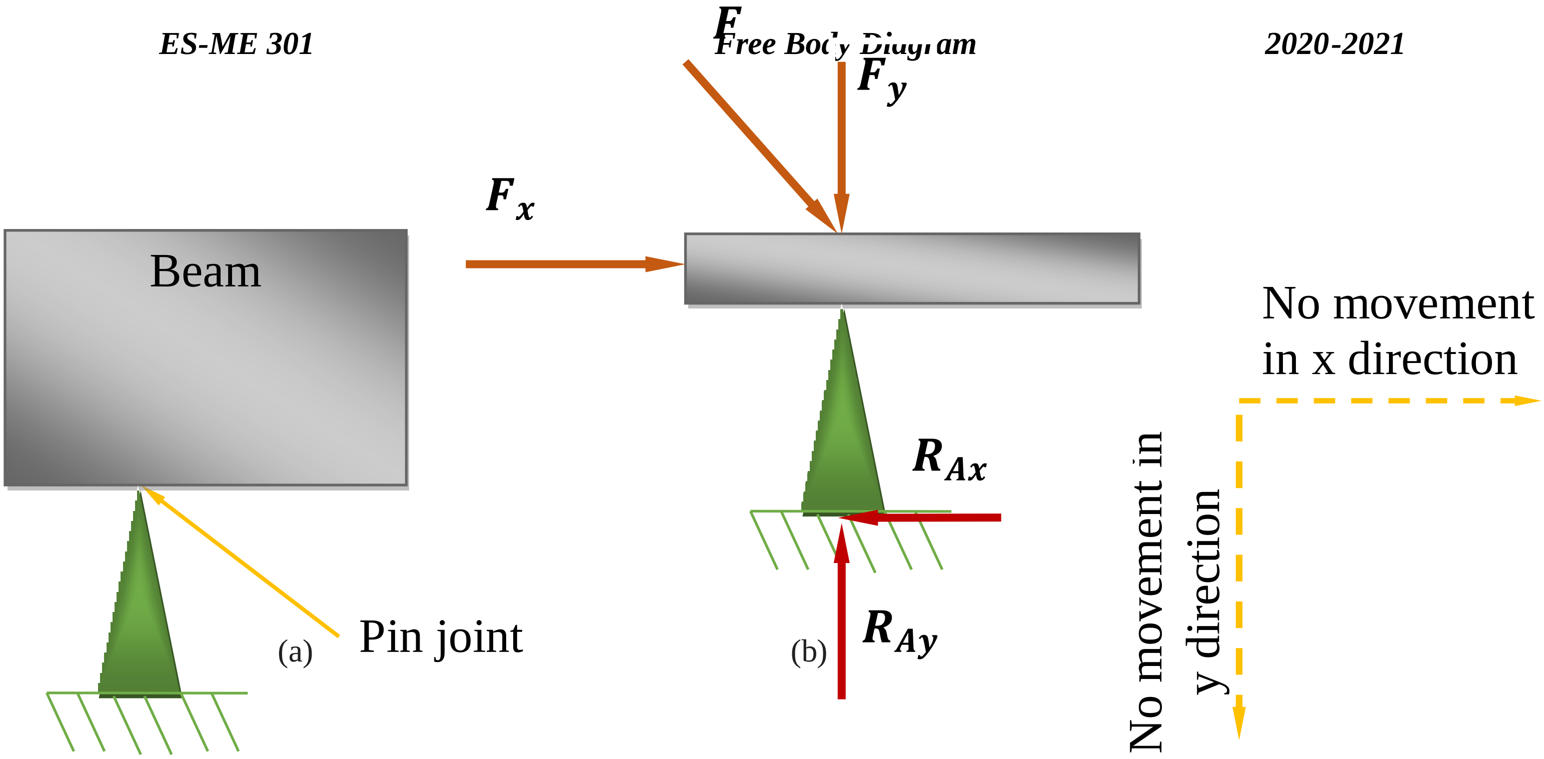



Fig. 8 Pinned support: (a) schematic presentation and (b) FBD

For better understanding a human elbow or knee can be considered as pinned joint. You can rotate your leg one direction but you can’t translate it with respect to knee. To prevent this rotation a structure needs at least two pin joint ore one pinned and one roller joint. This joint and support is frequently used in trusses.

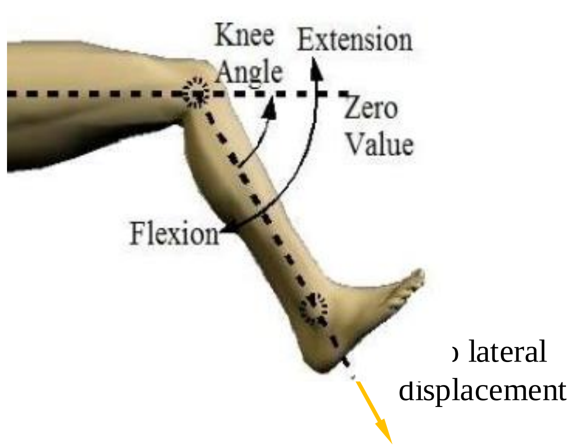


Fig. 9 Knee joint

## Fixed support

Fixed joints permit stability to certain areas of the body, although they do not move. Examples of fixed joints include the joints between the bones in the skull, Pillar. It is a rigid type of support or connection.

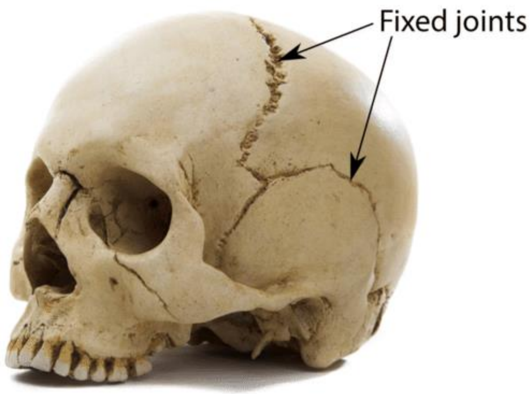
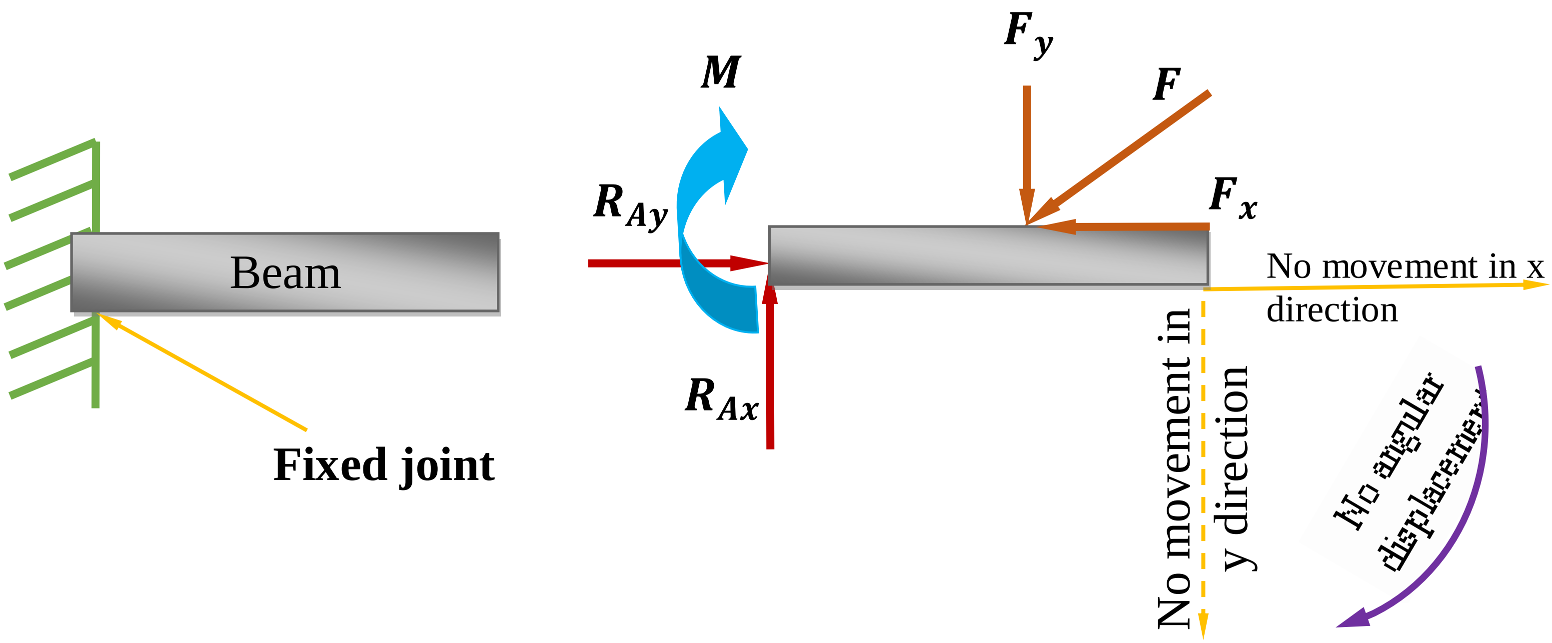



***Fig. 11 Joints between the bones in the skull and a vertical with ground***

The support connected with fixed joint is called the fixed support. In this support neither translation nor rotation is possible of the body. Here support reaction forces have both the components, Vertical and Horizontal in addition there is a moment M as it prevent rotation. The reaction force and moment is shown in Fig. 11.




(a) (b)

Fig. 12 Fixed Support: (a) schematic presentation and (b) FBD

## Simple support

Simple support is basically where the structural member rests on an external structure as shown in figure 11. This support is similar to roller support in a sense that it prevents vertical displacement but not horizontal. Therefore, it is not widely used in real life structures.

***Fig. 13 Simply supported structure***

Only vertical component of reaction $R_{Ay}$ is present there as shown in Figure 13.

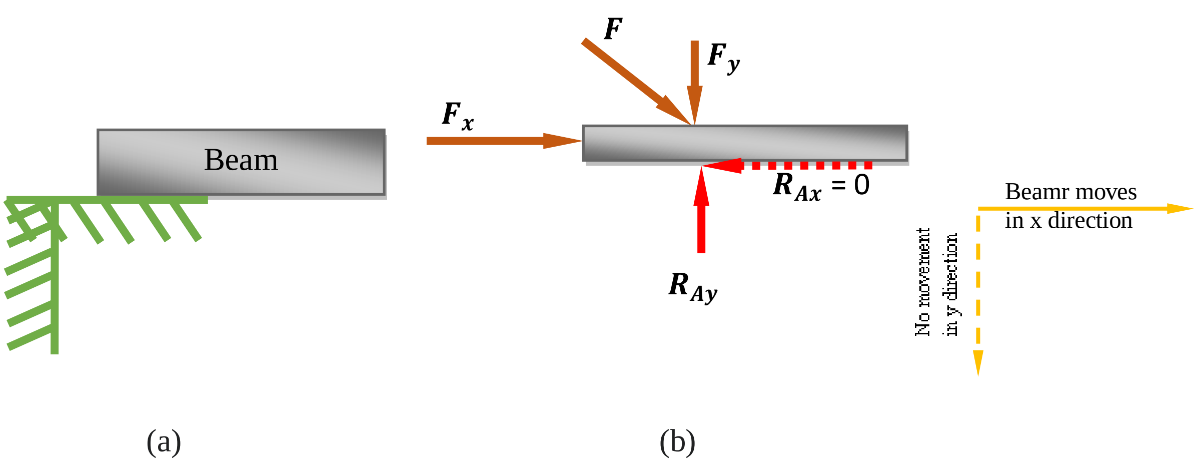
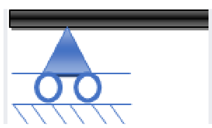
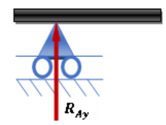
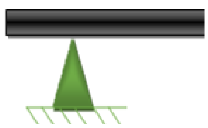
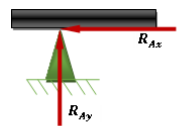
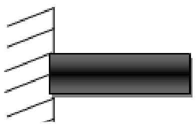
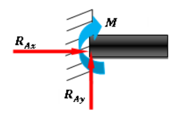
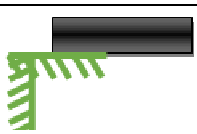
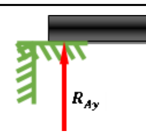



Fig. 12 Simple support: (a) schematic presentation and (b) FBD

Now let's summarize the these four joints, corresponding supports and the reactions working on them.

Type of Support

| **Type of Support** | **Schematic diagram** | **Allowed movement and constraints** | | | **Reaction** |
|---|---|---|---|---|---|
| | | Vertical | Horizontal | Rotation | |
| Roller Support |  | **No** | **Yes** | **Yes** |  |
| Pinned Support |  | **No** | **No** | **Yes** |  |
| Fixed Support |  | **No** | **No** | **No** |  |
| Simple Support |  | **No** | **Yes** | **Yes** |  |

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